

# Axiverse cosmology and the energy scale of inflation

David J. E. Marsh<sup>1,2\*</sup>, Daniel Grin<sup>3</sup>, Renée Hlozek<sup>4</sup>, and Pedro G. Ferreira<sup>5</sup>

<sup>1</sup>*Perimeter Institute, 31 Caroline St N, Waterloo, ON, N2L 6B9, Canada*

<sup>2</sup>*Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3RH, UK*

<sup>3</sup>*School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA*

<sup>4</sup>*Department of Astronomy, Princeton University, Princeton, NJ 08544, USA and*

<sup>5</sup>*Astrophysics, University of Oxford, DWB, Keble Road, Oxford, OX1 3RH, UK*

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Ultra-light axions ( $m_a < 10^{-18}$  eV), motivated by string theory, can be a powerful probe of the energy scale of inflation. In contrast to heavier axions the isocurvature modes in the ultra-light axions can coexist with observable gravitational waves. Here it is shown that large scale structure constraints severely limit the parameter space for axion mass, density fraction and isocurvature amplitude. It is also shown that radically different CMB observables for the ultra-light axion isocurvature mode additionally reduce this space. The results of a new, accurate and efficient method to calculate this isocurvature power spectrum are presented, and can be used to constrain ultra-light axions and inflation.

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*Introduction*— Axions [1] are a leading candidate for the dark matter (DM) component of the Universe. Proposed to solve the strong  $\mathcal{CP}$  problem, they are also generic in string theory [2], leading to the idea of an *axiverse* [3]. The number of axions in the axiverse is expected to be large, due to the topological complexity of string compactifications.

In the axiverse the axion mass is given by  $m_a^2 = \Lambda_a^4 / f_a^2$ . The scales,  $f_a$  and  $\Lambda_a$ , are both determined separately for each axion, and depend on the action,  $S$ , due to non-perturbative physics on the corresponding cycle:  $f_a \sim M_{pl}/S$ ,  $\Lambda_a^4 = \mu^4 e^{-S}$ , where  $M_{pl}$  is the reduced Planck mass,  $M_{pl}^2 = 1/8\pi G$ . The hard non-perturbative scale is  $\mu$  and its value should be roughly given by the geometric mean of the Planck scale and the SUSY scale. Solving the strong  $\mathcal{CP}$  problem with one of the string axions requires  $S \gtrsim 200$  [2, 3], giving rise to stringy values of  $f_a \gtrsim 10^{12}$  GeV, near the GUT scale. The exact value of  $S$ , however, scales with the area of the corresponding cycle (a dynamically distributed quantity in the landscape), so that small variations in the area lead to exponential variations in the scale of the potential. Thus the mass also depends exponentially on the modulus determining  $S$ , which is evenly distributed, and so the axion mass spectrum can be taken as an flat distribution on a logarithmic scale [3].

An important distinction between QCD axions and lighter axions is that the temperature dependence of the mass drops out and this changes the scalings between misalignment angle and relic density.

If axions are very light, with mass  $m_a \lesssim 10^{-18}$  eV (ultra-light axions, or ULAs), coherent oscillations of the field lead to the suppression of small scale power, and

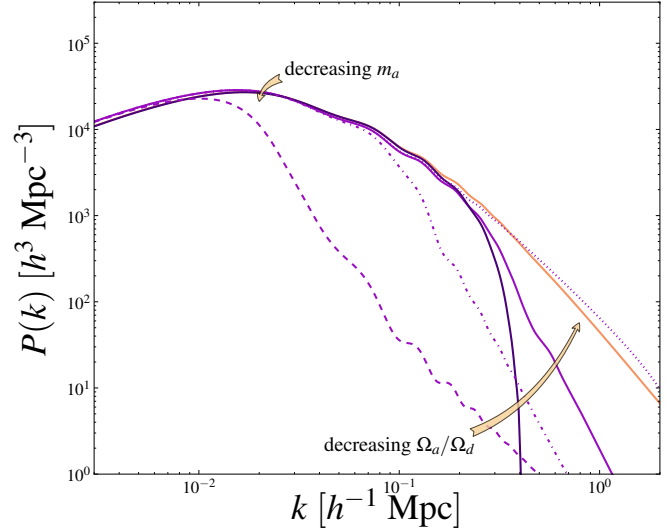


FIG. 1: Adiabatic matter power spectra, with varying axion mass  $m_a = 10^{-28}, 10^{-26}, 10^{-25}, 10^{-23}$  eV at fixed density fraction  $\Omega_a/\Omega_d = 0.5$  (dashed), and varying  $\Omega_a/\Omega_d = 0.1, 0.5, 1$  at fixed  $m_a = 10^{-25}$  eV (solid). Spectra are calculated using the methods of Ref. [14].

distinguish axions from cold (C)DM [4–6]. Heuristically, this scale is the geometric mean of the axion de Broglie wavelength and the Hubble scale. Depending on the axion mass, this scale could be macroscopic, thus affecting observed CMB anisotropies and galaxy clustering power spectra. For the classic QCD axion ( $m_a \gtrsim 10^{-6}$  eV), this scale is not cosmologically relevant. In the WKB approximation (averaging over the fast time scale associated with the axion mass,  $m_a^{-1}$ ), the axion may be accu-

\*dmarsh@perimeterinstitute.ca

rately treated as a fluid, with sound speed

$$c_a^2 = \begin{cases} \frac{k^2}{4m_a^2 a^2} & \text{if } k \ll 2m_a a, \\ 1 & \text{if } k \gg 2m_a a. \end{cases} \quad (1)$$

The scale of structure suppression begins at the scale  $k_m$ , defined such that those modes with  $k > k_m$  had sound speed  $c_a^2 = 1$  for some time while they were inside the horizon [6]. The effect saturates at the smaller scale  $k_J = \sqrt{m_a H}$ . Therefore, like massive standard model neutrinos or more novel warm (W)DM candidates [7], axions exhibit suppressed structure on small scales, as shown in Fig. 1.

Axions in inflationary cosmology carry *isocurvature* fluctuations [8], further distinguishing them from thermally produced CDM. The amplitude of these fluctuations is set by the energy scale of inflation and is thus tied to the amplitude of primordial gravitational waves. Axions thus offer an interesting window into the inflationary epoch, the string landscape, and the multiverse [11]. For a QCD axion, the isocurvature bounds imply that tensor modes are unobservably small: surprisingly this does not happen for ultra-light axions, and we will shortly explain why.

In past work, these two aspects of axion cosmology – the suppression of clustering and the existence of isocurvature perturbations – have been viewed in isolation. Probes for axion-seeded isocurvature have been restricted to the QCD axion, for which structure suppression on small scales is observationally irrelevant [11, 12]. Meanwhile, observations of CMB anisotropies and galaxy clustering place limits to axion-induced structure suppression [5], but do not yet include the isocurvature constraint.

*Isocurvature perturbations, gravitational waves and the CMB*– It is well known that the tensor-to-scalar ratio,  $r$ , is a probe of the inflationary energy scale and may be measured using CMB B-modes [9]. The isocurvature amplitude of axions is directly related to  $r$ , because both ULAs and gravitons are massless during inflation. The standard formulae for the tensor,  $\mathcal{P}_h$ , and scalar,  $\mathcal{P}_\mathcal{R}$ , power give the well known result for the tensor-to-scalar ratio  $r \equiv \mathcal{P}_h/\mathcal{P}_\mathcal{R} = 16\epsilon$ , where  $\epsilon$  is a slow roll parameter. Given that the scalar amplitude,  $A_s = (1/2\epsilon)(H_I/2\pi M_{pl})^2$ , is well measured, a measurement of  $\epsilon$  constitutes a measurement of the Hubble scale during inflation,  $H_I$ . The isocurvature fraction also depends on  $\epsilon$ : measuring it constrains a *function* of  $H_I$  and the axion initial misalignment angle [8]. To measure  $H_I$  using isocurvature one must either constrain or make assumptions about the axion initial misalignment angle. Tensor modes only measure  $H_I$  if they have an inflationary origin. The sensitivity to tensor modes and isocurvature improves with results from *Planck* [10].

Isocurvature perturbations are entropy fluctuations of the form  $S_{ij} = (\delta n_i/n_i) - (\delta n_j/n_j)$ , where the  $\delta n_i$  and  $n_i$  are number density fluctuations and average number densities, respectively, in each species present. Entropy fluctuations arise if there are (nearly) massless spectator

fields present during inflation [13], and the axion is one example.

Since the axion is an independent quantum field from the inflaton, energetically subdominant during inflation, the axion seeds isocurvature perturbations that are *uncorrelated* with the dominant adiabatic fluctuations. The axion isocurvature fluctuations generated in this manner are unavoidable in any standard inflationary scenario as long as neither the inflationary fluctuations of the axion nor reheating restore the Peccei-Quinn symmetry [11]. This is often the case for the large, stringy, values of  $f_a \gtrsim 10^{12} \text{GeV}$ .

The de Sitter space quantum fluctuations of the axion field  $\phi$ , have magnitude<sup>1</sup>:

$$\sqrt{\langle \delta\phi^2 \rangle} = \frac{H_I}{2\pi}. \quad (2)$$

There are constraints (e.g. WMAP9 [12]) on the relative amplitude,  $\alpha$ , of CDM isocurvature fluctuations defined by:

$$\frac{\alpha}{1 - \alpha} \equiv \frac{\mathcal{P}_\mathcal{S}(k_0)}{\mathcal{P}_\mathcal{R}(k_0)}, \quad (3)$$

where  $\mathcal{P}_\mathcal{S}$  is the isocurvature primordial power spectrum evaluated at pivot wavenumber  $k_0$ .

The axion power spectrum is given by:

$$\langle \delta_{a,i}^2 \rangle \approx 4 \left\langle \left( \frac{\delta\phi}{\phi} \right)^2 \right\rangle = \frac{(H_I/M_{pl})^2}{\pi^2 (\phi_i/M_{pl})^2}, \quad (4)$$

$$\left( \frac{\phi_i}{M_{pl}} \right)^2 \approx \frac{6H_0^2 \Omega_a}{m_a^2 a_{\text{osc}}^3}. \quad (5)$$

The initial misalignment angle is  $\theta_i = \phi_i/f_a$ : it is fixed by the relic density and  $a_{\text{osc}}$ , which is a function of axion mass defined by  $3H(a_{\text{osc}}) = m_a$  [6, 14]. Subsequent to  $a_{\text{osc}}$  the axion redshifts as matter, but displays suppression of structure formation.

Before we can relate  $\alpha$  to  $m_a$ ,  $H_I$  and  $\Omega_a$  using Eqs. (4)-(5), we must clarify the isocurvature normalisation. The usual CDM isocurvature normal mode is defined by taking  $\delta_c = 1$  as the the initial amplitude of the CDM overdensity, and normalizing the power spectrum such that  $\mathcal{P}_\mathcal{S} = \mathcal{P}_c$ , where  $\mathcal{P}_c$  is the power spectrum of the CDM fractional overdensity.

If axions are now included as a sub-component of the DM with the same equation of state and sound speed as CDM (as in Ref. [12] and others), then there is a single DM effective fluid (with fractional density perturbation  $\delta_d$ ) whose isocurvature normal mode is defined by  $\delta_d = 1$ . If axions carry isocurvature fluctuations, while the CDM itself carries only adiabatic fluctuations, then  $\mathcal{P}_\mathcal{S} =$

<sup>1</sup> These fluctuations also set the variance on initial misalignment angle and may alter the axion abundance  $\Omega_a$  [11]. In our mass range of interest this effect is negligible.



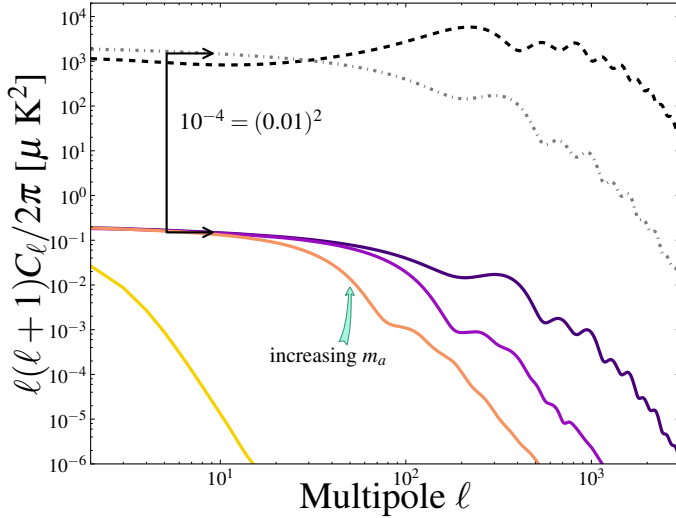


FIG. 3: CMB axion isocurvature power spectrum, with adiabatic  $\Lambda$ CDM for scale (black dashed). We demonstrate the normalisation difference between  $\alpha_{\text{CDM}}$  (grey dot-dash) and  $\alpha_a$  (solid), with  $\Omega_a/\Omega_d = 0.01$  implying a normalisation difference of  $(0.01)^2 = 10^{-4}$ . We also show small-scale power suppression by the lightest axions. The axion masses are  $m_a = 10^{-32}, 10^{-29}, 10^{-28}, 10^{-20}$  eV.

scale structure in ULA models, is carefully implemented using a modified version of CAMB [17] and is described in Ref. [14]. In this case, all other species fall into the gravitational potential wells set up by axions, and so axions drive the behavior of the observables, leading to far more dramatic effects. We show example spectra in Fig. 3.

Fig. 3 demonstrates that in the isocurvature mode, CMB power is suppressed on small scales (large  $\ell$ ), with the scale of power suppression becoming larger as the axion mass decreases, just as in  $P(k)$  (c.f. Fig. 1). As the axion mass increases the axion isocurvature spectra asymptote to CDM-like behaviour.

The suppression of power will be important for ULAs in altering the isocurvature constraints. Since the isocurvature power spectrum falls off rapidly at large  $\ell$ , most constraining power on isocurvature comes from the addition of power along the low- $\ell$  plateau before the first peak at  $\ell \sim 200$ . When the isocurvature power is suppressed along this plateau the isocurvature spectrum re-

mains significant only at lower and lower  $\ell$ . Therefore we should expect that not only will allowed values of  $\alpha_a$  be different from  $\alpha_{\text{CDM}}$  due to normalisation, but also due to the power suppressing properties of ULAs. The effect of this is estimated from the reduced number of modes available to measure isocurvature fraction and is shown in Fig. 2. Isocurvature becomes harder to measure and further constrains the observable region for  $\{\alpha, r\}$  at the lowest masses,  $m_a \lesssim 10^{-28}$  eV. The lowest mass region is harder to access observationally using LSS measurements since the structure suppressing properties of the axions only occur on very large scales [6]. In addition, producing an observable relic density with  $m_a \lesssim 10^{-28}$  eV would require additional physics: for example a large number of axions with nearly degenerate masses.

*Conclusions*— In this letter we have demonstrated that in the case of ultra-light axions one is able to unambiguously infer the energy scale of inflation from their isocurvature fraction by using large scale structure constraints to bound the relic density. In addition, there are regions of parameter space allowed by current constraints where both the isocurvature fraction and the tensor to scalar ratio are within observable reach of near future CMB experiments. This predicted concordance of three observables is a potentially powerful probe of the energy scale of inflation. In the context of the axiverse, the inferred value of  $H_I$  from observed tensor modes would predict observable axion isocurvature across more than four orders of magnitude in axion mass. We present constraints to this model in a forthcoming paper [14].

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